Matching Pursuit - a method of evaluation and parametrisation of non-stationary signals and transients

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Abstract: We have introduced to the analysis of physiological signals a new method called Matching Pursuit[1] [2], which adaptively fits to the signal waveforms chosen from a very large vocabulary of functions. The method was applied to simulated signals and sleep EEG. The signal components were clearly resolved in time-frequency coordinates and parametrised. The sleep spindles were identified and described in terms of occurrence in time and frequency, energy and time span. Comparison between automatic and visual detection of sleep spindles revealed high sensitivity and accuracy of the method. Matching Pursuit is especially well suited for analysis of non-stationary signals and transients which elude conventional methods of signal analysis.

INTRODUCTION

Most of the methods of EEG analysis applied up to now such as Fourier Transform or autoregressive models provided the information about the general frequency characteristics of the time series but did poorly in recognizing transients and non-stationary signals. A progress in respect of evaluation of this kind of signals was introduced by wavelet analysis, but the method has also some serious limitations. The most important of them is the fact that the bandwidth changes in steps and therefore expansion coefficients in a wavelet frame do not provide precise frequency estimates whose Fourier transform is well localized, especially at high frequencies. The representation is sensitive to a shift in time of analyzed window. Therefore wavelet transform is well suited to the analysis of time - locked phenomena such as evoked potentials [3] [4], but does poorly for transients occurring more or less randomly in the signal (e.g. sleep spindles, K complexes, epileptic spikes). Matching Pursuit (MP) chooses waveforms in such a way as to match at best local signal structures.

METHOD

Matching Pursuit (MP) method was introduced by Mallat and Zhang in 1993 [5]. In MP approach a family of time-frequency waveforms called also "atoms" is generated by scaling, translating and modulating a window function $g(t)$:

$$g_I(t) = \frac{1}{\sqrt{s}}\, g\left(\frac{t-u}{s}\right) e^{i\xi t} \qquad (1)$$

where $s>0$ is scale, ξ - frequency modulation and u - translation. Index $I=(s,\ \xi,\ u)$ describes the set of parameters. The windowed Fourier transform and wavelet transform can be considered as particular cases of MP corresponding to restrictions concerning the choice of parameters.

In the first step of the iteration procedure the function g_{I_0} is chosen which gives the biggest product with signal $f(t)$. Then the residual vector R_1 obtained after approximating f in the direction g_{I_0} is decomposed in similar way. The iterative procedure is repeated on the following obtained residues:

$$R^n f = \langle R^n f,\ g_{I_n}\rangle\, g_{I_n} + R^{n+1} f \qquad (2)$$

In this way the signal f is decomposed into a sum of time-frequency atoms, that are chosen to match at best its residues:

$$f = \sum_{n=0}^{\infty} \langle R^n f,\ g_{I_n}\rangle\, g_{I_n} \qquad (3)$$

For the visualization of signal's energy density in time-frequency plane we can define a distribution $Ef(t,\omega)$, which conserves signal's energy over the time-frequency plane:

$$Ef(t,\omega) = \sum_{n=0}^{+\infty} |\langle R^n f, g_{I_n}\rangle|^2\, W g_{I_n}(t,\omega) \qquad (4)$$

W - the Wigner distribution of a function $f(t)$ - is defined as

$$Wf(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t+\tau)\, \bar{f}(t-\tau)\, e^{-i\omega\tau}\, d\tau \qquad (5)$$

RESULTS

MP procedure was applied to 21 channels (10/20 system) of whole night EEG (sampling frequency 102.4 Hz). Gabor functions were used as basic waveforms. An example of Wigner plot of the 20 s segment of a signal is shown in Fig.1.

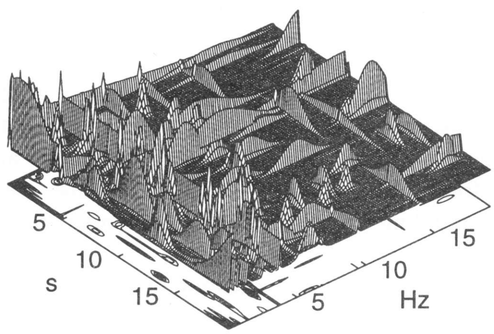

Figure 1.
3-dimensional representation of Wigner map obtained by means of MP for the 20s of EEG from sleep stage 2, derivation C3-A2. Height proportional to the $[Ef(t,\omega)]^{0.3}$ (eq. 4).

One can observe the lack of the cross-terms which disturb the spectra in the case of Wigner de-Ville transform. Each of the atoms is characterized by the following parameters: occurrence in time, frequency, time span, energy, phase. The identification of certain type of structure can be achieved by setting limits on these parameters. In case of sleep spindles we have chosen atoms characterized by frequency in the band 12-15 Hz and time span 0.6 - 2.4 sec.

We have performed comparison of sleep spindles detection by MP with human judgment. The number of false positives decreased with the amplitude, which demonstrates higher sensitivity of MP. Some structures classified by expert as one spindle were identified by MP as a superposition of spindles with different frequencies. The description of spindles in terms of above mentioned parameters made possible construction of different types of distributions and relationships. E.g. 1) Occurrence of spindles during whole night sleep (all spindles or spindles of given time span or frequency range or amplitude can be chosen). 2) Topographical (for 21 derivation) occurrence of spindles depending on frequency. 3) Tracing single spindles across derivations. The results of 1) showed that occurrence of sleep spindles is correlated with sleep stages, especially well visible was their absence during REM sleep. The results of 2) showed the predominance of low-frequency spindles in frontal and high frequency spindles in parietal and occipital derivations. Amplitudes of low frequency spindles were higher in frontal and lower in parietal and occipital derivations for high frequency spindles the situation was reversed.

DISCUSSION

MP overcomes a limitation inherent to wavelet transform, concerning the relation between time and frequency scales. MP offers a unique possibility of non-stationary signals representation in time-frequency-energy coordinates and identification/parametrisation of transients. It has been shown [5] that when Gabor functions are used as basic waveforms MP gives time-frequency resolution close to the theoretical limit. The spectral characteristics of short-time transients can be determined with high accuracy. Different types of functional relationships and distributions can be constructed on the basis of output parameters of the method. We have demonstrated the advantages of the method on the example of sleep spindles, but a method is by no means limited to this type of structures.

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